Nelineární odhad stavu při chybějících měřeních

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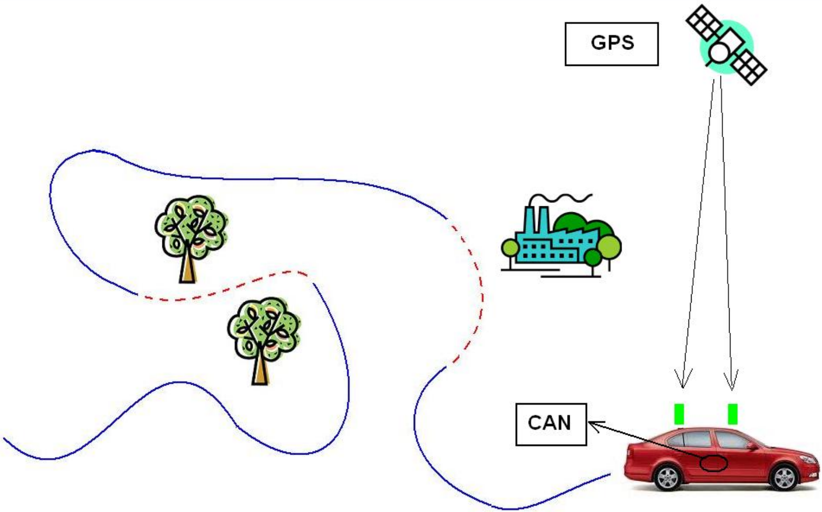
Motivation



State-space model

For $t \in t^* = 1, 2, \ldots, T$, $k \in k^* \subset t^*$

$$\begin{aligned} \mathbf{x}_t &= g(\mathbf{x}_{t-1}) + \mathbf{w}_t; \qquad f(\mathbf{w}_t) = \mathcal{U}(\mathbf{0}, \mathbf{p}) \\ \mathbf{y}_k &= h(\mathbf{x}_k) + \mathbf{e}_k; \qquad f(\mathbf{e}_t) = \mathcal{U}(\mathbf{0}, \mathbf{r}) \end{aligned}$$

- m-dimensional state $\mathbf{x}_t$;
- n-dimensional output $\mathbf{y}_k$;
- real vector function, $g : \mathcal{R}^m \rightarrow \mathcal{R}^m$;
- real vector function, $h : \mathcal{R}^m \rightarrow \mathcal{R}^n$;
- state and output noises $\mathbf{w}_t$ and $\mathbf{e}_t$;

Proposed model + estimation

Joint pdf

For $t \in t^* = 1, 2, \ldots, T,\ k \in k^* \subset t^*$

$$f(\mathbf{d}^{1:OUT:T}, \mathbf{x}^{0:T}, \Theta) \propto \left[\prod_{i=1}^{m} p_i\right]^{-T} \left[\prod_{j=1}^{n} r_j\right]^{-K} \chi(\mathcal{S})$$

$$\mathcal{S} : \begin{cases} \text{prior information} \\ \text{state-space model} \\ \text{restriction on states} \end{cases}$$

Proposed model + estimation

The MAP estimate of $\mathbf{X} = \left[\begin{array}{ccc} (\mathbf{x}^{0:t})' & \mathbf{p}' & \mathbf{r}' \end{array} \right]'$

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X} \in \mathcal{S}} \left(\sum_{i=1}^{m} \ln(p_i) + \frac{K}{T} \sum_{j=1}^{n} \ln(r_j) \right)$$

The MAP estimate $\rightarrow$ non-linear programming form

- missing data = missing inequality
- off-line estimate
- data at the beginning and at the end of data outage

GPS/INS data – usage in the model

quantity (unit)	notation	source	in model as
true position No. 1 (m)	(p_{x1}, p_{y1})	-	state
true position No. 2 (m)	(p_{x2}, p_{y2})	-	state
true azimuth (rad)	φ	-	state
true velocity (m/s)	v	-	state
true yaw rate [rad/s]	ω	-	state
measured position No. 1 (m)	$(\tilde{p}_{x1}, \tilde{p}_{y1})$	GPS	output
measured position No. 2 (m)	$(\tilde{p}_{x2}, \tilde{p}_{y2})$	GPS	output
measured azimuth (rad)	$\tilde{\varphi}$	GPS	output
measured velocity (m/s)	$\tilde{v}$	INS	output
measured yaw rate [rad/s]	$\tilde{\omega}$	INS	output

State model of moving vehicle

State equation:

$$
\begin{aligned}
p_{x1;t} &= p_{x1;t-1} + h v_{t-1} \sin \varphi_{t-1} && + w_{x1;t} \\
p_{y1;t} &= p_{y1;t-1} + h v_{t-1} \cos \varphi_{t-1} && + w_{y1;t} \\
p_{x2;t} &= p_{x2;t-1} + h v_{t-1} \sin \varphi_{t-1} && + w_{x2;t} \\
p_{y2;t} &= p_{y2;t-1} + h v_{t-1} \cos \varphi_{t-1} && + w_{y2;t} \\
\varphi_t &= \varphi_{t-1} - h\,\omega_{t-1} && + w_{\varphi;t} \\
v_t &= v_{t-1} && + w_{v;t} \\
\omega_t &= \omega_{t-1} && + w_{\omega;t},
\end{aligned}
$$

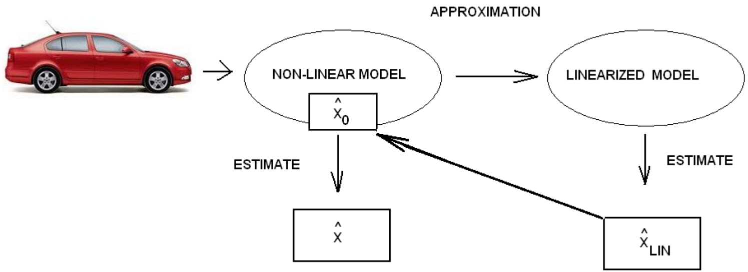
State model of moving vehicle

Output equation:

$$
\begin{aligned}
\mathcal{I}_t\ \tilde{p}_{x1;t} &= \mathcal{I}_t(p_{x1;t} + e_{x1;t}) \\
\mathcal{I}_t\ \tilde{p}_{y1;t} &= \mathcal{I}_t(p_{y1;t} + e_{y1;t}) \\
\mathcal{I}_t\ \tilde{p}_{x2;t} &= \mathcal{I}_t(p_{x2;t} + e_{x2;t}) \\
\mathcal{I}_t\ \tilde{p}_{y2;t} &= \mathcal{I}_t(p_{y2;t} + e_{y2;t}) \\
\mathcal{I}_t\ \tilde{\varphi}_t &= \mathcal{I}_t(\xi(p_{x1;t}, p_{y1;t}, p_{x2;t}, p_{y2;t}) + e_{\varphi;t}) \\
\tilde{v}_t &= v_t + e_{v;t} \\
\tilde{\omega}_t &= \omega + e_{\omega;t}
\end{aligned}
$$

$\mathcal{I}_t \in 0, 1$ - measurement indicator

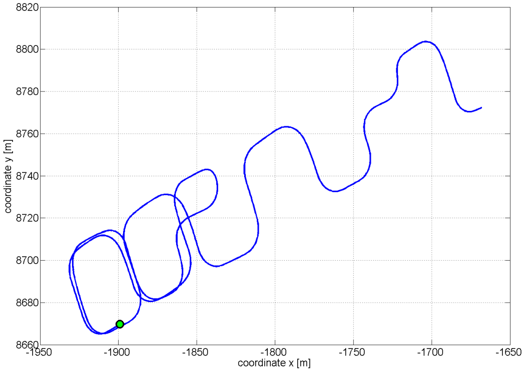
Problem of initial setting



Linearised model

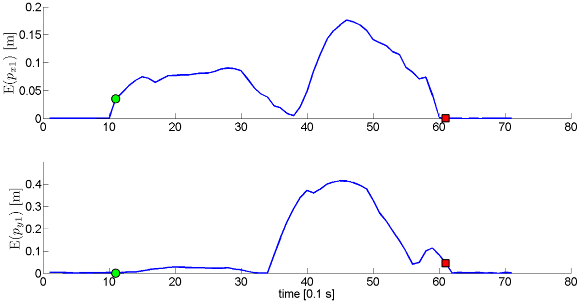
- states v, ω $\longrightarrow$ inputs
- $\hat{\varphi} = \begin{cases} \tilde{\varphi}_t & \dots \quad \text{if GPS data are available} \\ \hat{\varphi}_{t-1} - h\omega_{t-1} & \dots \quad \text{otherwise} \end{cases}$

Experiments



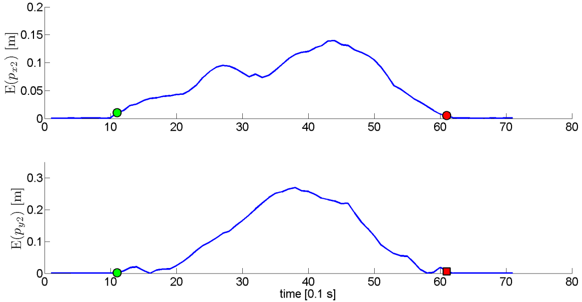
- real data from training place
- simulated outages in complete GPS data
- result evaluation - absolute error of estimates

Experiments



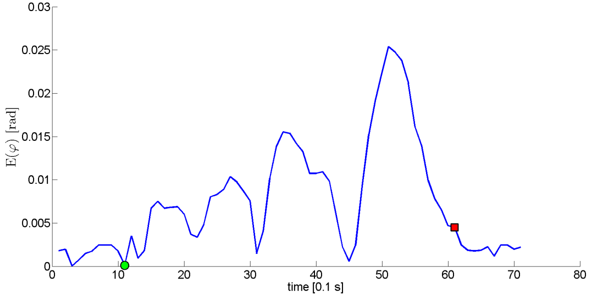
Obrázek: Course of absolute error for estimates $p_{x1;t}$, $p_{y1;t}$

Experiments



Obrázek: Course of absolute error for estimates $p_{x2;t}$, $p_{y2;t}$

Experiments



Obrázek: Course of absolute error for estimates φ_t

Summary

- linear $\longrightarrow$ nonlinear model
- choice of more appropriate quantities
- definition of states, inputs, outputs
- Gaussian $\longrightarrow$ uniform noise