Fully Probabilistic Design

and

Preference Elicitation

M. Kárný and many others

who try to understand it :(

Introduction

FPD uses ideal model for expressing user's aims.

In comparison to LQG control it has plus (+) and minus (-):

$+$

- more general
- demands on the whole closed loop can be expressed

$-$

- setting of the ideal is not easy

Formulation

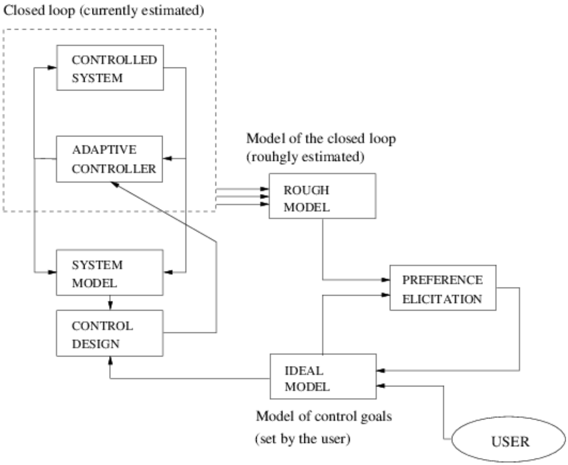
The task is to perform on-line setup of the ideal model, based on the currently evaluated behavior of the closed loop.

This behavior is described by a rough model. This model is roughly estimated from roughly measured data.

Rough = with larger period than basic period of sampling.

The block scheme of the situation follows: $\downarrow$

Scheme



Closed loop (currently estimated)
CONTROLLED SYSTEM
ADAPTIVE CONTROLLER
SYSTEM MODEL
CONTROL DESIGN
Model of the closed loop (rouhgly estimated)
ROUGH MODEL
PREFERENCE ELICITATION
IDEAL MODEL
Model of control goals (set by the user)
USER

Closed loop

At time t, the model of the closed loop is

$$f\left(d_t | \phi_{t-1}\right)$$

where

$d_t = \{y_t,\, u_t\}$ is the current data item (y_t output, u_t input)

ϕ_{t-1} is a vector of old data items on which y_t depends

It can be factorized

$$f\left(d_t|\phi_{t-1}\right) = \underbrace{f\left(y_t|u_t, \phi_{t-1}\right)}_{\text{system model}} \underbrace{f\left(u_t|\phi_{t-1}\right)}_{\text{controller}}$$

Uninteresting outputs

Example: When modeling car consumption, we have

- inputs: gas, break, gear
- outputs: consumption, speed, moment, revs

There is no reason for penalizing moment and revs = uninteresting outputs

Model of the closed loop

$$
\begin{aligned}
f\left(d_t|\phi_{t-1}\right) &= f\left(y_t^n|y_t^i, u_t, \phi_{t-1}\right)\, f\left(y_t^i|u_t, \phi_{t-1}\right) f\left(u_t|\phi_{t-1}\right) = \\
&= f\left(y_t^n|y_t^i, u_t, \phi_{t-1}\right)\, \underbrace{f\left(u_t|u_t, \phi_{t-1}\right) f\left(y_t^i|\phi_{t-1}\right)}_{f\left(y_t^i, u_t|\phi_{t-1}\right)}
\end{aligned}
$$

Models

- Model of the controlled system $\cdots \quad f(\cdot|\cdot)$
  - *used for adaptive controller*
- Rough model of the closed loop $\cdots \quad f^R(\cdot|\cdot)$
  - *behavior of the closed loop*
- Ideal model of the closed loop $\cdots \quad f^I(\cdot|\cdot)$
  - *desired behavior of the closed loop*

FPD + uninteresting outputs

Rough model

$$f^R(d_t|\phi_{t-1}) = f^R\left(y_t^n|y_t^i, u_t, \phi_{t-1}\right)\ f^R\left(y_t^i|u_t, \phi_{t-1}\right) f^R(u_t|\phi_{t-1})$$

Ideal model

$$f^I(d_t|\phi_{t-1}) = f^I\left(y_t^n|y_t^i, u_t, \phi_{t-1}\right)\ f^I\left(y_t^i|u_t, \phi_{t-1}\right) f^I(u_t|\phi_{t-1})$$

Minimization of

$$KL\left(\prod f^R | \prod f^I\right)$$

$\rightarrow$

FTP result

$$f\left(u_t|\phi_{t-1}\right) = \frac{f^I\left(u_t|\phi_{t-1}\right)\exp\left\{-\varsigma\left(u_t,\phi_{t-1}\right)\right\}}{\gamma\left(\phi_{t-1}\right)}$$

where

$$\varsigma\left(u_t,\phi_{t-1}\right) = E\left[\ln\frac{f\left(y_t^i|y_t^n,u_t,\phi_{t-1}\right)}{f^I\left(y_t^i|y_t^n,u_t,\phi_{t-1}\right)\gamma\left(\phi_t\right)}\middle|u_t,\phi_{t-1}\right] \quad \leftarrow$$

$$\gamma\left(\phi_{t-1}\right) = \int_{u^*} f^I\left(u_t|\phi_{t-1}\right)\exp\left\{-\varsigma\left(u_t,\phi_{t-1}\right)\right\}du_t$$

ς only from y^i (y^n is canceled)

Elicitation task formulation

Control problem to be solved:

- setpoint following for interesting outputs

$$\int y_t^i \, f^I \left(y_t^i | \phi_{t-1}\right) dy_t^i = \int y_t^s \, f^R \left(y_t^s | \phi_{t-1}\right) dy_t^s = \bar{y}_t^s$$

- conservative controller - not to move the behavior of the closed loop too far from the existing one

Setpoint following

- the request for setpoint following concerns only y^i - the expectation is

$$E\left[y_t^i|\phi_{t-1}\right] = \int y_t^i\, f^I\left(y_t^i|\phi_{t-1}\right) dy_t^i$$

i.e. it concerns the marginal $f^I\left(y_t^i|\phi_{t-1}\right)$

- the corresponding factorization is

$$f^I\left(y_t^i, u_t|\phi_{t-1}\right) = \underbrace{f^I\left(u_t|y_t^i, \phi_{t-1}\right)}_{\text{non causal controller}} \underbrace{f^I\left(y_t^i|\phi_{t-1}\right)}_{\text{marginal in } y_t^i}$$

- $f^I\left(y_t^i|\phi_{t-1}\right)$ is chosen from f^R and with expectation $\bar{y}_t^s$ we denote it by

$$f^{OI}\left(y_t^i|\phi_{t-1}\right) \text{ Optimistic Ideal}$$

($f^I\left(u_t|y_t^i, \phi_{t-1}\right)$ is still free)

Conservative controller

- the last term of the ideal model that is to be determined is $f^I\left(u_t | y_t^i, \phi_{t-1}\right)$
- under condition of minimum KL distance between ideal and rough models, the result is

$$f^I\left(u_t | y_t^i, \phi_{t-1}\right) = f^R\left(u_t | y_t^i, \phi_{t-1}\right)$$

which is obtained from the reverse factorization of the rough model

Example

Simulation

- 2 dimensional dynamic 2nd order regression model with constant

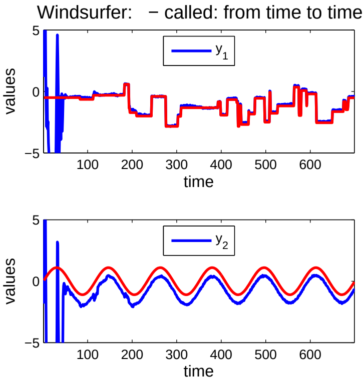
Filtration

- data normalization
- no structure estimation

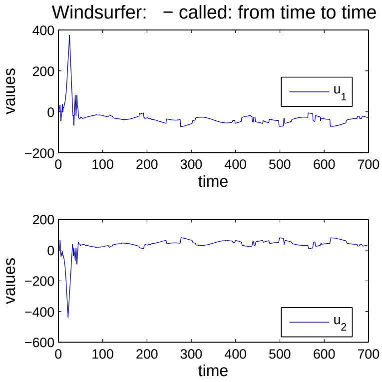
Rough model

- the same structure as system model + static controller
- period for estimation = 10
- practically no limitation for inputs

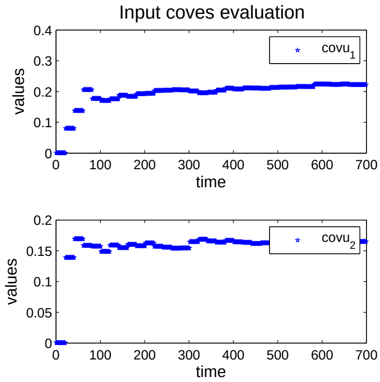
System outputs



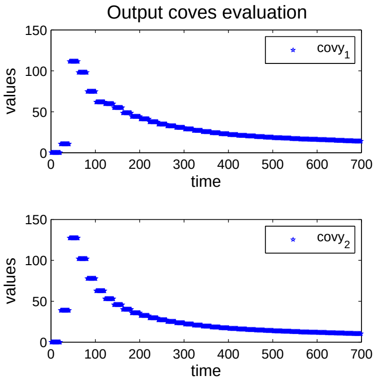
System inputs



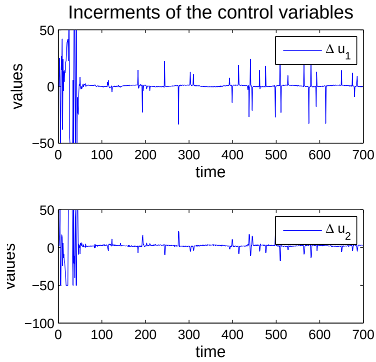
Input variances



Output variances



Increments of control variables



Conclusions

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